

Steering Geometry and Caster Measurement

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ABSTRACT

Caster of a steerable vehicle wheel is defined herein to be relative to the thrust line of the non-steerable wheels. This incorporates caster into the “total alignment” concept. A method of measuring caster in accordance with this definition is derived. The restrictions, limitations, and accuracy of the method are investigated, and practical implementation procedures are suggested.

The purpose of this paper is to define the caster angle of a steerable vehicle wheel to be referenced to the thrust line of the non-steerable wheels. A further purpose is to derive and characterize an optimal method of measuring caster which can be implemented in a practical manner.

Recent years have seen the development of the “total alignment” concept, which relates the toe angles of the steerable vehicle wheels to the thrust line of the nonsteerable wheels. Increased sophistication of vehicle suspension systems has made this more important, while advances in alignment measurement instrumentation have incorporated the concept and assumed the corresponding computational burden.

Referencing caster to the thrust angle is part of the total alignment concept, but caster of a steerable wheel is difficult to measure. It is the angle between the vertical and the projection of the invisible steering axis onto a vertical plane containing the thrust line of the vehicle. It is not easy to attach measuring devices to a projection of an invisible axis. Indirect measurement methods are available that are very accurate, if they are implemented properly and certain restrictions are observed.

REFERENCING CASTER TO THE THRUST LINE

Caster has been defined as follows:

“6.2.1 Caster Angle — The angle in side elevation between the steering axis and the vertical. It is considered positive when the steering axis

is inclined rearward (in the upward direction) and negative when the steering axis is inclined forward.” [1] *

Positive caster tends to produce a stable steering system by generating counterbalancing torques about the steering axes as the wheels roll. The torques vary as the steer (toe) angles change. An equilibrium condition exists when the steer angles of the wheels remain constant with no driver-applied torque to the steering wheel. Ideally, the front wheels steer the vehicle in a straight line in this “neutral steer” condition.

Left and right caster must be equal for this condition to occur, other factors being equal. However, the straight line direction of travel is the thrust line of the nonsteerable rearwheels. If the thrust angle of the rear wheels is altered, the neutral steer direction of the front wheels no longer coincides with the thrust line. See Figure 1. The vehicle then rolls in a circle unless the driver steers the front wheels by applying a torque to the steering wheel. The vehicle “pulls” to the side.

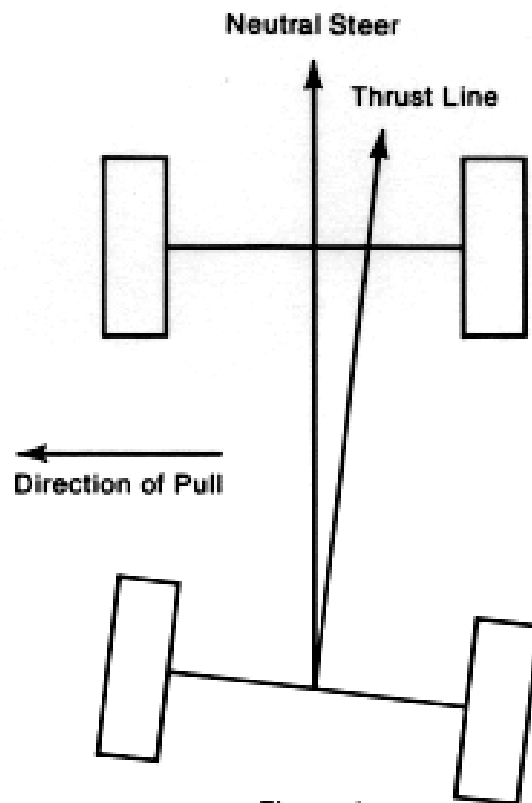


Figure 1

If the neutral steer direction does not coincide with the thrust line, the vehicle pulls to the side.

*Numbers in brackets designate references at end of paper.

Clearly caster must be defined relative to the thrust line. This is easily done:

Caster Angle — The angle, in side elevation *parallel to the thrust line of the non-steerable wheels*, between the steering axis and the vertical. It is considered positive when the steering axis is inclined rearward (in the upward direction) and negative when the steering axis is inclined forward.

This improved definition brings caster measurement in accord with the total alignment concept, where individual toe angles are referenced to the thrust line of the rearwheels.

CASTER MEASUREMENT METHODOLOGY

Caster cannot be measured directly, since one cannot mount a sensor on an imaginary steering axis. Instead, caster can be computed from changes in camber as the wheel toe angle is changed. The methodology is easily derived.

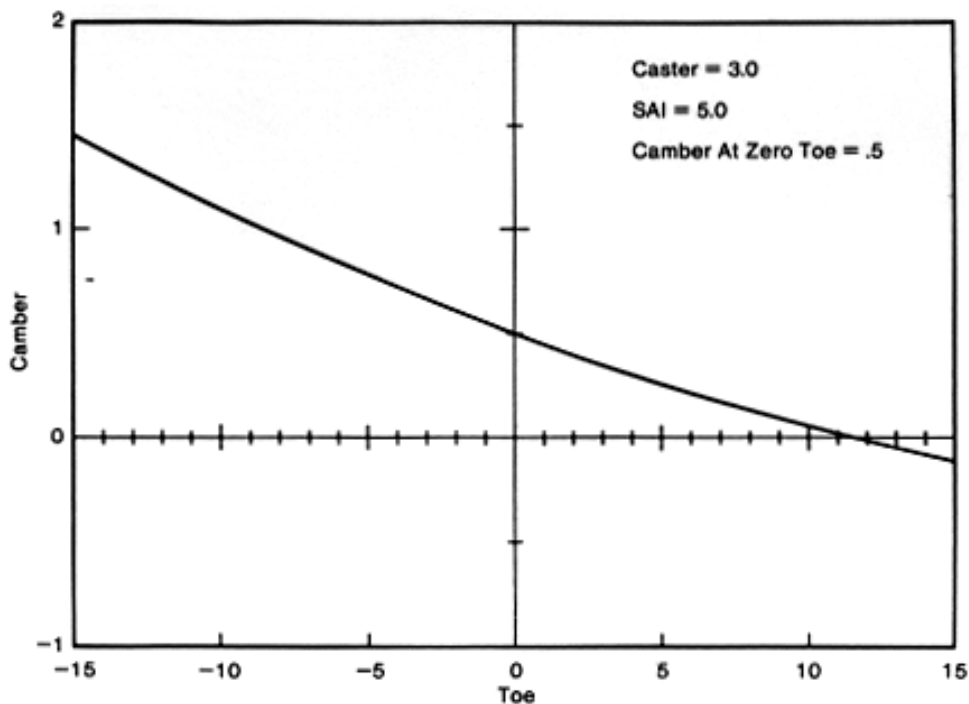


Figure 2

The variation in camber as the wheel is steered is determined by caster, SAI, and camber at zero toe.

The camber of a steerable wheel is determined by the caster, steering axis inclination (SAI), toe angle, and the camber angle at zero toe. The equation describing this relation (derived in Appendix A) is as follows:

$$\sin C = (\cos C_0 - \cos C \cos T) \tan S - \cos C \sin T \tan K + \sin C_0 \quad (1)$$

where

C = camber

C_0 = camber at zero toe

K = caster

S = SAI

T = toe (relative to the thrust line)

The variation in camber as the toe angle changes is illustrated in Figure 2.

The caster measurement procedure is to steer the wheel to two toe angles T_1 and T_2 where respective camber measurements C_1 and C_2 are made. Applying (C_1, T_1) and (C_2, T_2) to Eq. (1) yields

$$\sin C_1 = (\cos C_0 - \cos C_1 \cos T_1) \tan S - \cos C_1 \sin T_1 \tan K + \sin C_0 \quad (2)$$

$$\sin C_2 = (\cos C_0 - \cos C_2 \cos T_2) \tan S - \cos C_2 \sin T_2 \tan K + \sin C_0 \quad (3)$$

Subtracting Eq. (3) from Eq. (2) gives

$$\begin{aligned} \sin C_1 - \sin C_2 &= (\cos C_2 \cos T_2 - \cos C_1 \cos T_1) \tan S \\ &\quad + (\cos C_2 \sin T_2 - \cos C_1 \sin T_1) \tan K \end{aligned} \quad (4)$$

Solving for caster yields

$$\begin{aligned} K = \tan^{-1} \left(\frac{\sin C_1 - \sin C_2}{\cos C_2 \sin T_2 - \cos C_1 \sin T_1} \right. \\ \left. - \frac{\cos C_2 \cos T_2 - \cos C_1 \cos T_1}{\cos C_2 \sin T_2 - \cos C_1 \sin T_1} \tan S \right) \end{aligned} \quad (5)$$

Note that Eq. (5) is independent of camber at zero toe (C_0), which is highly desirable, since C_0 need not be measured to compute caster. Unfortunately, Eq. (5) is dependent on SAI (S), which is highly undesirable, since SAI is unknown. Dependence on SAI can be made negligible by making one approximation and one restriction in procedure.

The camber angles measured during this ‘‘caster turn’’ procedure are very small, usually under two degrees. The approximation $\cos C = 1$ is made, the error at two degrees being only 0.06%. Eq. (5) then reduces to

$$K \approx \tan^{-1} \left(\frac{\sin C_1 - \sin C_2}{\sin T_2 - \sin T_1} - \frac{\cos T_2 - \cos T_1}{\sin T_2 - \sin T_1} \tan S \right) \quad (6)$$

A restriction is made that the ‘‘caster turn’’ be symmetric about the thrust line, that is $T_2 = -T_1$. The usual procedure is to steer to the left to toe

angle T_1 and measure C_1 , then steer to the right to toe angle $T_2 = -T_1$ and measure C_2 . With this restriction, Eq. (6) reduces to

$$K \approx \tan^{-1} \left(\frac{\sin C_1 - \sin C_2}{\sin T_2 - \sin T_1} \right) \quad (7)$$

Eq. (7) allows caster to be *approximately* computed directly from toe and camber measurements, provided the toe and camber measurements comply with the restrictions discussed above. The computations required can be simplified even further, especially for use in analog instruments. If two further approximations are made. These are

$$\tan x \approx x \times (\pi/180)$$

$$\sin x \approx x \times (\pi/180)$$

where x is in degrees. These approximations are quite good at the small angles commonly encountered in measuring caster, and their errors tend to cancel when substituted in Eq. (7), which then simplifies to

$$K \approx \frac{180}{\pi} \times \frac{C_1 - C_2}{T_2 - T_1} \quad (\text{degrees}) \quad (8)$$

LIMITATIONS OF PROCEDURE

The procedure described above and using Eq. (8) to compute caster will measure caster relative to the thrust line of the vehicle if two requirements are met:

- (1) The toe angles T_1 and T_2 must be measured relative to the thrust line of the non-steerable wheels.
- (2) The caster turn must be symmetric about the thrust line, i.e. $T_2 = -T_1$.

If the turn is not symmetric, then the second term of the right hand side of Eq. (6) is not zero. In effect, this produces crosstalk between the measurements of SAI and caster by invalidating the assumptions that lead to Eq. (7) and Eq. (8).

This can be visualized quite easily. A more rigorous definition of caster is

Caster Angle — The angle, in a vertical longitudinal plane containing the thrust line, between the vertical and the projection of the steering axis onto the plane.

The caster turn measurement procedure measures the angle in the plane which bisects the total turn angle. If the thrustline does not coincide with the bisector of the turn, then the projection of the steering axis onto the plane is at a different angle from the vertical, and so is not the angle defined to be caster.

This can be intuitively understood in another way. Eq. (8) shows that caster is computed from a change in camber ($C_1 - C_2$) as the wheel is steered through a turn angle ($T_2 - T_1$), assuming $T_2 = -T_1$. If the turn is offset so as to be the same total amount but asymmetric, then the change in camber will be different, because camber does not vary proportionately to toe, as Figure 2 shows.

What practical effect does turn asymmetry have on caster adjustment? If caster is measured by an asymmetric turn procedure and adjusted to be "equal" on both wheels, it will not be equal on both wheels relative to the thrust line. This may generate a pull to the side if the side-to-side caster difference is significant and the suspension is sensitive to this difference.

The theoretical accuracy of measuring caster using Eq. (8) is shown in Figures 3 and 4. Figure 3 illustrates the error in measured caster as a function of actual caster for various amounts of turn asymmetry. Figure 4 illustrates the error in measured caster as a function of caster for various combinations of SAI and camber at zero toe, assuming a symmetric turn. It is obvious from these figures that the method is theoretically quite accurate, provided the restrictions in procedure are met.

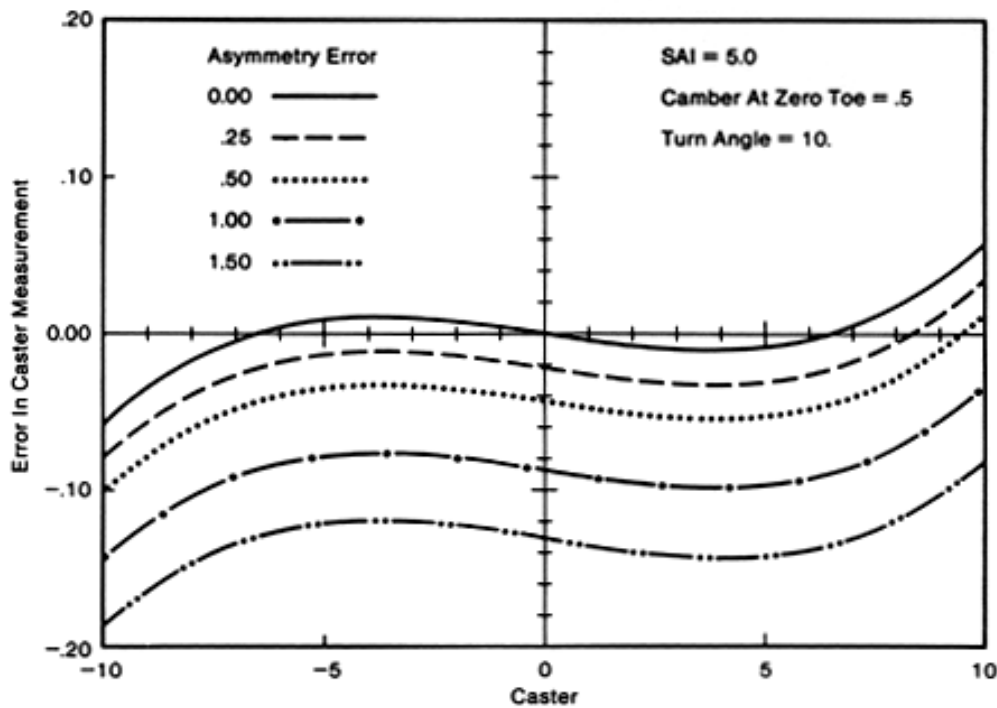


Figure 3

Caster measurement error varies with turn asymmetry.

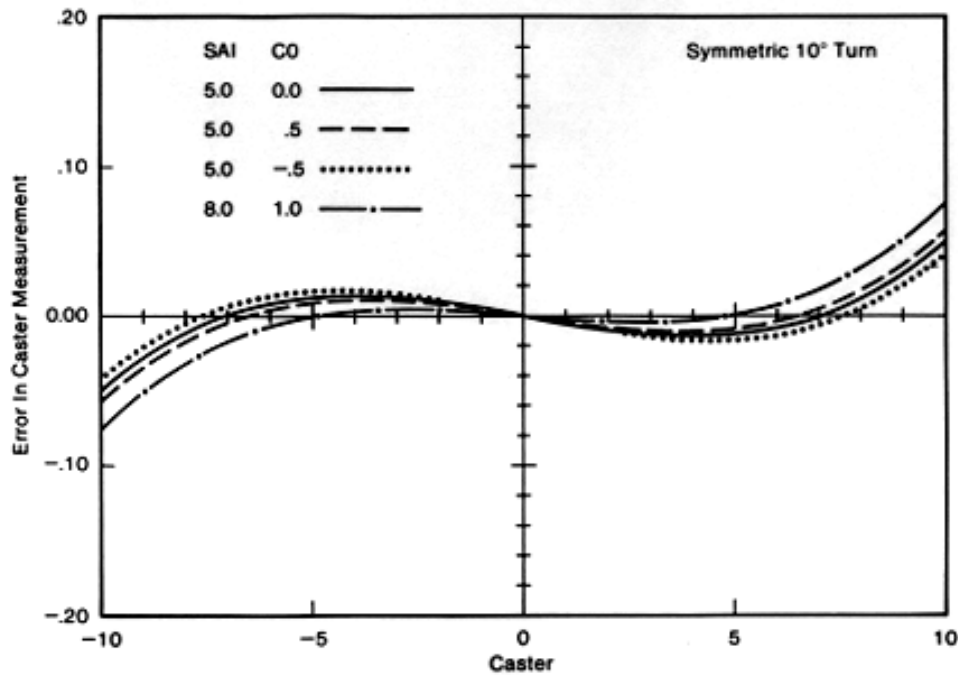


Figure 4

Caster measurement is relatively insensitive to SAI and camber at zero toe.

PRACTICAL CASTER MEASUREMENT

A practical implementation of this method of measuring caster must adhere to the restrictions enumerated above. This is most easily accomplished using a microcomputer and electronic toe and camber sensors. The sensors must be capable of measuring camber and toe during the caster turn procedure, and toe must be measured relative to the thrust line.

The usual procedure begins with steering a wheel to the left to a specified toe angle, usually -10° to -15° , and measuring and storing both camber and toe (C_1 and T_1). The wheel is then steered to the right to the opposite toe angle ($T_2 = -T_1$) where camber and toe are again stored (C_2 and T_2). Finally, caster is computed using Eq. (8).

Notice that this procedure measures the caster of only one wheel, the one which is steered to the proper toe angles. If the left wheel is steered to the proper angles but camber and toe are stored and caster computed for both left and right wheels, the right caster can be expected to be in error, since the turn of the right wheel can be expected to be asymmetric. The asymmetry is due to the difference in turn angles of the two wheels (toe-out-on-turns) and due to total toe, which is usually not zero. Figure 3

illustrates the seriousness of this asymmetry error.

This asymmetry error can be eliminated by steering the left wheel to the correct angles to measure left camber and left toe, then steering the right wheel to the correct angles to measure right camber and right toe. The procedures can be mixed by steering the left wheel to the left, the right wheel to the left, the left wheel to the right, and the right wheel to the right. Obviously, other variations are also possible.

A potential error source in this type of steering procedure is hysteresis in the wheel suspension during the turn. This can be caused, for example, by compliance in rubber suspension parts, in tires, and in the turnplates on which the tires rest. An optimum procedure is to steer too far to the left, then steer toward the right to the proper left turn angle and measure camber and toe, then steer toward the right to the proper right turn angle and measure camber and toe. For example, steer to the left to -15° , then to the right to -10° and measure camber and toe, then steer to the right to $+10^\circ$ and measure camber and toe. Recall that caster is computed from the *change* in camber during this steering procedure. In practice, if one steer angle is approached from the left while the other is approached from the right, the camber change measured during the turn will be in error due to suspension hysteresis. For example, if camber is offset only 0.05° due to hysteresis, the error in measured caster is

$$\begin{aligned} \text{Caster error} &= \frac{180}{\pi} \times \frac{0.05}{10 - (-10)} \quad (\text{degrees}) \\ &= 0.14^\circ \end{aligned} \quad (9)$$

Certainly this error should be avoided if possible.

Note also that the sensors must have substantial resolution and accuracy to measure caster properly, especially when their outputs are digitized. If the turn is made to -10° and $+10^\circ$, Eq. (8) becomes

$$\begin{aligned} K &= \frac{180}{\pi} \times \frac{C_1 - C_2}{10 - (-10)} \quad (\text{degrees}) \\ &= 2.8648... \times (C_1 - C_2) \end{aligned} \quad (10)$$

The resolution of caster is approximately one third the resolution of camber, thus showing the need for accurate, high resolution camber sensors when measuring caster. Fortunately, this computation method does not require accurate zero calibration of the camber sensor. Since only the change is measured, only the range calibration need be accurate.

PRACTICAL STEERING DURING THE CASTER TURN PROCEDURE

During the caster turn procedure, the alignment technician is required to steer to somewhat precise toe angles and cause the alignment instrument to record the camber and toe angles. The intelligence of a microcomputer can be put to good use in directing and automating this activity.

For example, consider the caster measurement procedure used with the C111 Alignment System manufactured by Hunter Engineering Co. The console directs the steer operations by the use of “bar graph” indicators which direct the technician to steer to the proper angle by turning the steering wheel until a moving pointer is centered or “pulled” on a horizontal scale. When the bar graph is pulled, the wheel is steered to the proper angle, within a small tolerance.

The technician begins by instructing the console to measure caster, which responds by directing the technician to steer the wheels to an approximately straight ahead position. At this point the console records the offset between the front longitudinal toe sensors and the individual toe angles of the respective front wheels, so that the front toe angles can be measured during the turn procedure using only these longitudinal sensors. This is necessary because the line-of-sight of the transverse sensors might be blocked during the turn by large diameter tires.

The console then presents two bar graphs, one for each front wheel, which direct the technician to steer 10° to the left, within a tolerance of $\pm 1/4^\circ$. When a wheel is steered correctly and the sensors' signals are not changing, the console records the camber and toe angles of that wheel, and turns off the pointer of the bar graph. The process is then repeated with the other bar graph and the other wheel. The technician may steer the left wheel correctly first, then the right, or vice-versa, or both may be steered correctly at the same time (provided the steering geometry and total toe make it possible).

When both wheels have been steered correctly to the left and measurements have been recorded, the technician is directed to repeat the procedure by steering 10° to the right, within the same tolerance of $\pm 1/4^\circ$. After recording the camber and toe readings of both wheels while steered to the right, the console displays only one bargraph to direct the technician to steer approximately straight ahead, at which point the caster angles are computed and displayed.

By directing the technician to steer to within $1/4^\circ$ of the correct angle, the turn asymmetry is held to a maximum of $1/4^\circ$. Notice that the actual steer

angles are measured and used to compute caster, thereby keeping errors to a minimum.

STEERING USING TURNPLATE GAUGES

It is possible to measure caster by less sophisticated apparatus, but a compromise in accuracy and turn symmetry must be accepted. A common method is to turn the wheels to the correct angle by watching the gauges of the turnplates on which the wheels rest, then actuate a switch to record the camber measurements.

There are several limitations of this procedure. The first is the possibility of offset between the scale of the turnplate and the toe angle of the wheel relative to the thrust line. A common procedure to minimize this offset is as follows:

- (1) Jack the front wheels up so they clear the turnplates.
- (2) Steer the front wheels straight ahead, so that they have equal toe relative to the thrust line.
- (3) Rotate the turnplates until their pointers match the toe angles.
- (4) Lower the front wheels and jounce.
- (5) Steer and measure caster.

A possible problem with this procedure is that the toe angles change as the wheels are raised, but the turnplates might not turn to match the wheels as they are lowered. The step of jacking and lowering the wheels is also highly undesirable from a practical point of view.

A second limitation of this procedure is that the technician must steer the wheel by pushing and pulling directly on the tire, instead of simply turning the steering wheel. Eliminating the effects of suspension hysteresis is quite difficult under this condition.

A third limitation is that the actual turn angles are not measured. The computations performed by the alignment instrument assume the technician steered to the correct angles. If camber is not measured at these assumed turn angles, the caster computed will be in error. Figure 5 illustrates the sensitivity of caster measurement to this error, assuming a symmetric turn.

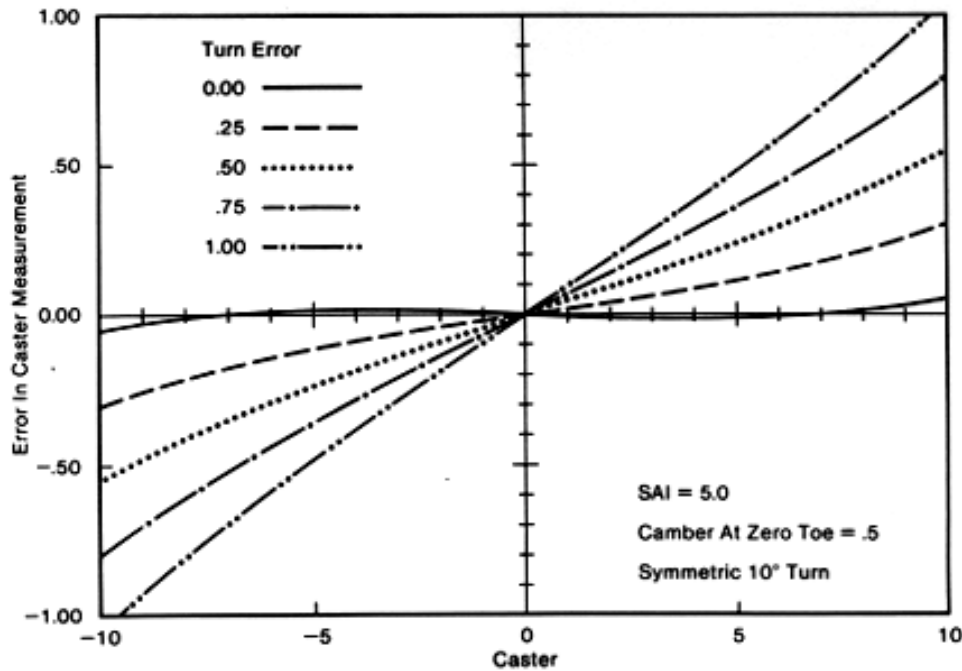


Figure 5

Caster measurements is sensitive to turn accuracy when turn angles are incorrect, even if the turn is symmetric.

These limitations can be somewhat overcome by instrumenting the turnplate, such that the console can correlate the turnplate angle with the actual toe angle, and thereby measure the actual turn angles. This also eliminates the undesirable step of jacking up the front wheels prior to measuring caster, but increases the expense, complexity, and maintenance requirements of the turnplate.

MECHANICAL SYSTEMS

Totally mechanical alignment instruments can easily implement this method of measuring caster, but are subject to the same requirements to produce accurate results. This type of instrument generally uses turnplates to guide the turn, and so suffers from the same limitations discussed in the previous section.

A typical method of measuring camber is to rotate a cam to level a bubble level and then read camber from a scale attached to the cam. Caster can be measured with this type of camber sensor if a proper caster scale is used. The procedure is to steer to the left to the correct angle, rotate the cam to level the bubble level, and rotate the caster scale to indicate zero.

Then steer to the right to the correct angle, level the bubble level, and read caster from the scale.

The caster scale is easily computed from Eq. (8). The procedure requires the technician to steer to predetermined angles T_1 and T_2 , so the scale factor is

$$\text{caster} = \frac{180}{\pi(T_2 - T_1)} \times \text{camber change} \quad (11)$$

STEERING AXIS INCLINATION (SAI)

SAI is defined in a manner similar to caster:

Steering axis inclination — The angle, in front elevation perpendicular to the thrust line of the nonsteerable wheels, between the steering axis and the vertical. It is considered positive when the steering axis is inclined inward (in the upward direction) and negative when the steering axis is inclined outward.

SAI is essentially an angle similar to caster, but is measured in a plane 90° to the plane in which caster is measured. SAI can thus be computed from the changes in a camber-type sensor mounted 90° to the usual camber sensor. (Such a sensor is normally used when adjusting caster.) The only extra requirement is that, during the turn procedure, the sensor assembly must be locked to the adapter which mounts it to the wheel, and the brakes must be locked. Such a sensor measures the same angular change a “camber” sensor would measure if it were mounted on a horizontal axle at 90° to the wheel axle.

In fact, both caster and SAI can be measured simultaneously during the caster turn procedure. The alignment instrument must have both a camber sensor and a “caster adjust” sensor, and the sensor and brakes must be locked, as described above. During the turn procedure, the camber sensor is tilted to the front or rear, and the angle it measures is altered by the cosine of the tilt angle. Since this angle is normally less than two degrees, the error introduced is virtually unmeasurable. The “caster adjust” sensor is tilted in the camber direction in the same manner, and the error here is also negligible.

SUMMARY

The definition of caster has been expanded such that it is referenced to the thrust line of the non-steerable wheels, thereby incorporating caster into

the “total alignment” concept. A method for measuring caster according to this definition has been derived, and the restrictions, limitations, and potential accuracy investigated. Practical procedures have been suggested for implementing this method, such that the alignment technician is guided through the procedure in accordance with the necessary restrictions.

ACKNOWLEDGEMENT

Special thanks are given to Mr. Dick Henry of General Motors Corp. for his closed form solution to Eq. (1), which is shown in Appendix B.

REFERENCES

- [1] “Vehicle Dynamics Terminology” SAE J670e, a report of the Vehicle Dynamics Committee, revised July 1976.

APPENDIX A — DERIVATION OF Eq. (1)

Of interest herein is the derivation of Eq. (1), which relates the camber of a steerable vehicle wheel to the caster and SAI angles of the steering axis to which it is attached, to the toe angle to which it is steered, and to the camber angle when toe is zero.

Refer to Figure 1A, which illustrates a steering axis with an attached axle. The following assumptions are made:

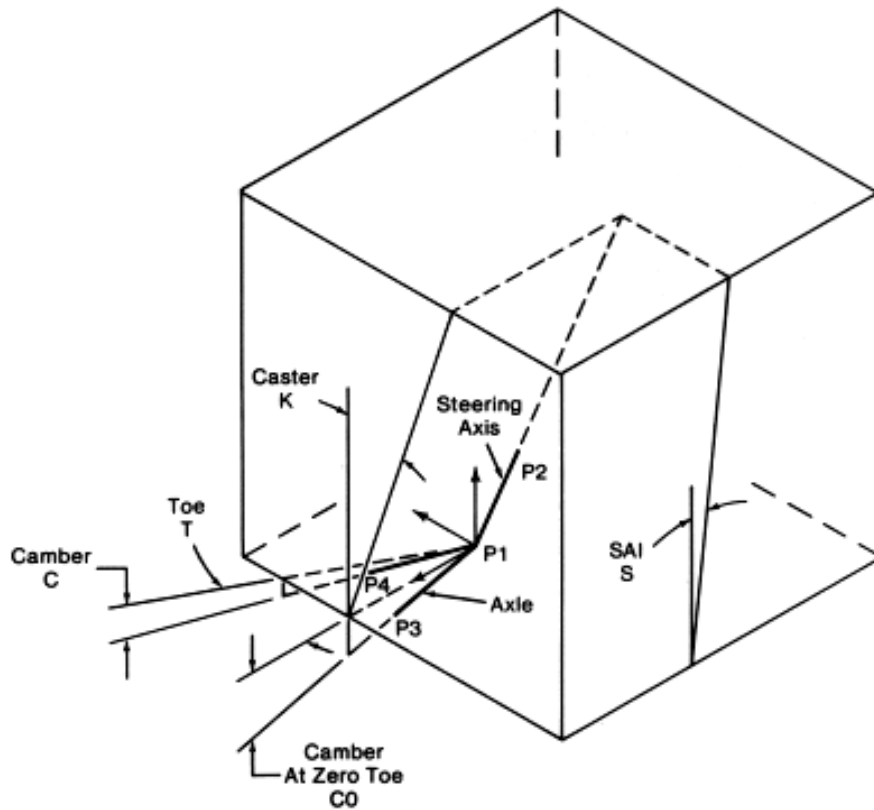


Figure 1A

A steering axis with attached axle illustrated the geometry of caster, SAI, camber and toe.

- (1) The axle-steering axis assembly is free to rotate about the steering axis.
- (2) The axle and steering axis each have unit length. (This greatly simplifies the discussion which follows.)
- (3) An orthogonal (X, Y, Z) coordinate system exists with the origin at the junction of the axle and the steering axis. The Y -axis is the thrust line of the vehicle. The Z -axis is vertical.

The following definitions are made:

- (1) K = caster = the angle between the Z -axis and the projection of the steering axis on the Y - Z plane.
- (2) S = steering axis inclination (SAI) = the angle between the Z -axis and the projection of the steering axis on the X - Z plane.
- (3) C = camber = the angle between the axle and the X - Y plane.
- (4) T = toe = the angle between the X -axis and the projection of the axle on the X - Y plane.
- (5) C_0 = camber when toe = 0.

Note that in Figure 1A all angles shown are positive.

The lower pivot point of the steering axis, point P_1 , is located at the origin:

$$P_1 = (0, 0, 0) \quad (1A)$$

Since the steering axis has unit length, the coordinates of the upper pivot point P_2 are established as follows:

$$X_2^2 + Y_2^2 + Z_2^2 = 1 \quad (2A)$$

$$\tan^{-1} Y_2/Z_2 = -K \quad (3A)$$

$$\tan^{-1} X_2/Z_2 = -S \quad (4A)$$

Substituting Eq. (3A) and Eq. (4A) into Eq. (2A),

$$Z_2^2 \tan^2 S + Z_2^2 \tan^2 K + Z_2^2 = 1 \quad (5A)$$

Solving for Z_2 :

$$Z_2 = 1/\sqrt{1 + \tan^2 K + \tan^2 S} \quad (6A)$$

Let $Z_2 = q$. Substituting into Eq. (3A) and Eq. (4A):

$$X_2 = -q \tan S \quad (7A)$$

$$Y_2 = -q \tan K \quad (8A)$$

Thus

$$P_2 = (-q \tan S, -q \tan K, q) \quad (9A)$$

One end of the axle is fixed at point P_1 . When the axle/steering axis assembly rotates about the steering axis, the other end of the axle moves in a circle. Since the axle has unit length, its movable end point at zero toe is at point P_3 , whose coordinates are easily found by inspection:

$$P_3 = (\cos C_0, 0, -\sin C_0) \quad (10A)$$

When the assembly is rotated to some toe angle T , the movable end point moves to point P_4 , where

$$\sin^{-1} Z_4 = -C \quad (11A)$$

$$\cos^{-1}(X_4/\cos C) = T \quad (12A)$$

$$\sin^{-1}(Y_4/\cos C) = T \quad (13A)$$

$$P_4 = (\cos C \cos T, \cos C \sin T, -\sin C) \quad (14A)$$

The angle between the steering axis and the axle remains constant during this rotation. Thus

$$\text{angle}[P_2, P_1, P_3] = \text{angle}[P_2, P_1, P_4] \quad (15A)$$

The angle between two lines in a three dimensional space can be found using direction cosines. The three direction cosines of a line are the cosines of the three angles between the line and the three axes. If a line passes through the origin and through point $P = (X_p, Y_p, Z_p)$, which is a unit distance from the origin, the direction cosines of the line are:

$$\cos D_x = X_p \quad (16A)$$

$$\cos D_y = Y_p \quad (17A)$$

$$\cos D_z = Z_p \quad (18A)$$

The angle between two such lines is found by

$$\cos(\text{angle}) = \cos D_{x1} \cos D_{x2} + \cos D_{y1} \cos D_{y2} + \cos D_{z1} \cos D_{z2} \quad (19A)$$

The angle $[P_2, P_1, P_3]$ can be found by substituting the coordinates of P_2 and P_3 into Eq. (19A).

$$\cos[P_2, P_1, P_3] = -q \tan S \cos C_0 - q \sin C_0 \quad (20A)$$

The angle $[P_2, P_1, P_4]$ is found in a similar manner

$$\cos[P_2, P_1, P_4] = -q \tan S \cos C_0 \cos T - q \tan K \cos C \sin T - q \sin C \quad (21A)$$

In light of Eq. (15A), Eq. (21A) is subtracted from Eq. (20A) and divided by q , giving

$$\tan S \cos C_0 + \sin C_0 = \tan S \cos C \cos T + \tan K \cos C \sin T + \sin C \quad (22A)$$

Rearranging

$$\sin C = (\cos C_0 - \cos C \cos T) \tan S - \cos C \sin T \tan K + \sin C_0 \quad (1)$$

This equation is valid for all combinations of K , S , T , and C_0 .

APPENDIX B — A CLOSED FORM SOLUTION TO Eq. (1)

Eq. (1) appears at first glance to be unsolvable for camber. A solution exists, however:

$$\sin C = (\cos C_0 - \cos C \cos T) \tan S - \cos C \sin T \tan K + \sin C_0 \quad (1)$$

Rearranging

$$\sin C + \cos C (\cos T \tan S + \sin T \tan K) - \cos C_0 \tan S + \sin C_0 = 0 \quad (1B)$$

Let

$$Q_1 = 1/(\cos T \tan S + \sin T \tan K) \quad (2B)$$

$$Q_2 = Q_1 (\cos C_0 \tan S + \sin C_0) \quad (3B)$$

Then

$$Q_1 \sin C + \cos C - Q_2 = 0 \quad (4B)$$

$$(Q_1 \sin C - Q_2)^2 = (-\cos C)^2 \quad (5B)$$

$$Q_1^2 \sin^2 C - 2Q_1 Q_2 \sin C + Q_2^2 = \cos^2 C \quad (6B)$$

$$= 1 - \sin^2 C \quad (7B)$$

$$(Q_1^2 + 1) \sin^2 C + (-2Q_1 Q_2) \sin C + (Q_2^2 - 1) = 0 \quad (8B)$$

Eq. (8B) is a quadratic in $\sin C$. Thus there are two possible roots. Let

$$a = Q_1^2 + 1 \quad (9B)$$

$$b = -2Q_1 Q_2 \quad (10B)$$

$$c = Q_2^2 - 1 \quad (11B)$$

One possible root is

$$\sin C = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad (12B)$$

The other is

$$\sin C = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad (13B)$$

Trial examples have shown that Eq. (12B) is valid if SAI is negative while Eq. (13B) is valid if SAI is positive. Both equations produce the same result if SAI is zero.

Iterative numerical methods also find roots quite well. The equation is very well behaved, allowing simple methods to converge quickly with high accuracy.

APPENDIX C — COMPUTATION OF FIGURES 2-5

Figure 2 is a plot of camber (C) as a function of toe (T) for a specified combination of caster (K), SAI, and camber at zero toe (C_0). This was directly computed using Eq. (12B) and Eq. (13B).

Figures 3 and 4 are plots of the errors in measured caster for various combinations of actual caster (K), SAI, camber at zero toe (C_0), and turn angles (T_1 and T_2). Figure 3 illustrates the sensitivity of the measurements to turn asymmetry, while Figure 4 illustrates the relative insensitivity to actual SAI and camber at zero toe when the turn is symmetric. The procedure to plot any given point is

- (1) Choose K , S , C_0 , T_1 , and T_2 .
- (2) Use Eq. (12B) or Eq. (13B) to compute C_1 and C_2 at T_1 and T_2 .
- (3) Use Eq. (8) to compute the measured caster.

(4) Plot (measured caster $-K$) vs. K .

Figure 5 is a similar plot, but measurements are made at turn angles T_3 and T_4 while caster is computed using different angles T_1 and T_2 . The computation procedure is

- (1) Choose K, S, C_0, T_1, T_2, T_3 , and T_4 .
- (2) Use Eq. (12B) or Eq. (13B) to compute C_1 and C_2 at T_3 and T_4 .
- (3) Use Eq. (8) with T_1 and T_2 to compute the measured caster.
- (4) Plot (measured caster $-K$) vs. K .